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A STATISTICAL ANALYSIS OF THE ROTATED SIGNS OF THE PHAISTOS DISC

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Abstract

The Phaistos Disc is an artifact from Phaistos, Crete (Greece), over 3000 years old. It is a circular disc with on each side a spiral with pictorial signs, mainly of humans, animals and plants. Although discovered over a century ago, its nature is still unclear.

Several signs of the Phaistos Disc are rotated; the various occurrences of such a sign do not have the same orientation. For instance one occurrence of a sign might be upside down compared with other occurrences of the same sign.

In this paper, in particular rotated signs are considered where the rotation seems not to be motivated by lack of space. The two most important rotated signs are the cat and the flying bird. The hypothesis is put forward that these rotations are made on purpose, rather than by mistake.

Firm statistical support for this hypothesis is given, based on the concentration among only two or three signs. This result was obtained with the Monte Carlo method, using millions of random replications. A comparison is made with an approximate analytical approach.

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The rotations did not yet receive attention as a significant aspect of the Disc. Hence this result might shed some new light on the nature of this ancient object and stimulate further research by archaeologists.

Introduction

The Phaistos Disc is an artifact from Phaistos, Crete (Greece), over 3000 years old and discovered during the excavation of a palace of the Minoan culture. It is a circular disc of clay, with on each side a spiral with pictorial signs. Although discovered over a century ago, its nature is still unclear.

There are 45 pictorial signs on the Disc, most of which occur more than once, giving a total of 241 occurrences. The signs are usually indicated by their number (1 to 45), and sometimes by a standard name describing the sign.

At several places along the two spiraling sequences of signs are dividing lines. These lines are said to separate the series of signs into “words”. Side A contains the words A1 to A31; side B contains words B1 to B30.

Most professional archaeologists who study the Disc consider it as a document, written in some unknown script in a possibly unknown language. They expect that it can be deciphered only after other examples of this script are found, which has not happened yet.

This paper discusses an aspect of the Disc which has received very little attention, namely the rotations of some signs. For instance, one occurrence of a sign might be rotated 180 degrees compared with other occurrences of the same sign.

Figure 1. Cat (29), Eagle (31) and Hide (27). Source: www.unicode.org[1]

[1]The Phaistos Disc occupies series 101D through 101F of the Unicode Standard 6.2. Our Figure 1 differs from this standard as follows. First, the pictures are mirrored along the vertical axis (swapping left and right); in this way, the resulting pictures are the same as on the Disc. (The convention of representing these signs in mirror image is due to the probable direction in which the Disc must be read: from right to left.) Second, the Cat (29) is rotated somewhat, to make the boundary line vertical, as discussed in the text.

The Rotations. Consider the six occurrences of sign 33 (Tunny) on the Phaistos Disc. They have a consistent orientation: each head points upwards. This is not so with all signs. We define a sign as rotated when there is a variation in the orientation of the occurrences of that sign. Note that a sign with only one occurrence cannot be rotated in this sense.

These rotations have received little attention; in particular the possibility that some of them are created on purpose (other than to save space). If this possibility turns out to be likely, it might aid our understanding of the nature of the Disc - either when the rotations are non-linguistic in nature, or when they have some presently unknown linguistic function.

Examples of rotated signs which are probably motivated by saving space are sign 02 (Plumed head) in the crowded word A29 and sign 03 (Tattooed head) in the narrow word A31.

Rotated signs *not* motivated by saving space are 29 (Cat, 11 occurrences) and 31 (Eagle, 5 occurrences). They have rotations of both 90 degrees and 180 degrees and they do not lack space around them.

The pair of signs 27 (Hide) in word A29 is rotated 180 degrees ("head down") compared with all other occurrences of this sign ("head up"). This might be motivated by saving space. See Duhoux [1], p. 24 (section 6A-8). See also his section 6A-10, about the possibility (suggested by someone else) that the rotation of the signs 27 in A29 is a mistake which implies a particular direction of the writing (from the centre outwards). He dismisses this particular line of reasoning as not logical. Also, this might be related to the sign 27 in word A23; see Cate [4]. We do not draw any conclusion here and proceed with sign 27 optionally.

The orientations of Cat (29) and Eagle (31). The orientations of the signs 29 and 31 are as follows:

• Sign 29 (11 occurrences) has its head to the right in words B19, B20 and B21; down in B29; to the left in A3 and B15; up in B18 and B26; in between up and left in B13; in between right and down in A4 (twice).

• Sign 31 (4 occurrences) has its head to the right in A16; up in A9 and A25; to the left in A22.

The above three orientations of sign 31 are obvious. However, this is not so with the above orientations of 29. These orientations are based on the following choice: an orientation of sign 29 is called “head to the right” if the line which forms the boundary between the head and the rest of the body is vertical, as in Figure 1.

Of course, the *differences* between the rotations of sign 29 would be the same with any other such choice. The signs 29 in A4 might alternatively be called “head down” (instead of “between right and down”). Then for instance the sign 29 in B29 would change from “down” to “between down and to the left”. The whole terminology would shift 45 degrees clockwise. The advantages of the choice actually made above are the easy visual identification of the orientations (using the boundary line of the neck) and the small number of “in between” orientations.

The concentration of these rotations (not motivated by lack of space) among only three signs (or two, excluding sign 27) of a total of 45 signs might be on purpose or coincidental. In order to analyze this, we set up the null hypothesis that they are coincidental.

The evaluation of this null hypothesis requires the number of *rotated occurrences* of the rotated signs discussed above. These occurrences are rotated compared to the unrotated, or “correct”, occurrences of that sign. Of course we do not know which are the unrotated occurrences, since rotation is a relative concept. Our choice of the unrotated occurrences is such that their number is as large as possible and hence the number of rotated occurrences is as small as possible. This minimizes the risk of a Type I error, i.e., rejecting the null hypothesis while in fact it is true. Note that a small number of rotated occurrences could more easily be coincidentally concentrated among only a small number of signs. This is particularly important since rejection of the null hypothesis might suggest a new line of inquiry in the century old search for the nature of this ancient artifact.

The smallest number of rotated occurrences. The smallest number of rotated occurrences among signs 27 and 29 and 31 is ten: two occurrences of sign 27 and six of sign 29 and two of 31. This is the result of the following choices of unrotated occurrences.

Signs 27: of course the two rotated signs 27 are the two in A29, with the 13 other occurrences considered unrotated.

Signs 29: the five unrotated signs 29 are the three with the head to the right and also the two undecided cases in A4. The latter have more or less their head to the right (according to our definition) and are included to find the lowest possible number of rotated signs.

Signs 31: the two unrotated signs 31 are those with head up. This might not be the natural position, flying up into the air, but any other choice gives a larger number of rotated occurrences.

Is the concentration of rotations among only a few signs a coincidence?

What is the probability that the rotations come from only a few signs? To take the case including sign 27: what is the probability that a random sample without replacement of 10 out of the 241 occurrences of all signs on the Disc comes from only three signs (or less)? This requires the probability distribution of the number of rotated signs under the null hypothesis.

This distribution is studied below with (a) an analytic approximation, the Occupancy model, and (b) a numerical method, Monte Carlo, which is precise to any desired degree.

The Occupancy Model

Before computing this distribution with a Monte Carlo simulation, let us first approximate the distribution using an analytic model: the classical Occupancy model. In this model, k balls are randomly put into n cells, the cells having equal probability. The probability distribution of the number of empty (not "occupied") cells is as follows:

$$\Pr(z|k, n) = \binom{n}{z} \sum_{m=0}^{n-z} (-1)^m \binom{n-z}{m} \left(1 - \frac{z+m}{n}\right)^k,$$

where z is the number of empty cells (zero balls). See for instance Ma [3] and the reference therein to Feller [2].

In order to apply this model to our problem, assume that for each sign the number of occurrences is large enough to accommodate all rotations. Then the Occupancy model tells us the probability of k rotations being randomly distributed over $n = 45$ signs and leaving z cells empty. In our case, with $k < n$, the above formula is only valid for $z \geq n - k$.

First, it is shown that if the k rotations are concentrated at only one sign, the formula simplifies to $(1/n)^{k-1}$, as one would expect:

$$\Pr(z = n-1 | k, n) = \binom{n}{n-1} \sum_{m=0}^{1} (-1)^m \binom{1}{m} \left(1 - \frac{n-1+m}{n}\right)^k$$

$$= n\left[\left(1-\frac{n-1}{n}\right)^k - \left(1-\frac{n-1+1}{n}\right)^k\right]$$

$$= n\left[\left(\frac{1}{n}\right)^k - 0\right] = \left(\frac{1}{n}\right)^{k-1} = \left(\frac{1}{45}\right)^9 \approx 10^{-15}.$$

We have $k = 10$ rotations when we include those of sign 27.

Next, applied to our problem, we get the total probability of the rotations being concentrated at three signs or less is approximately $10^{-15} + 10^{-11} + 10^{-8} \approx 10^{-8}$, where the first term was derived above. This is a very small probability, at any standard.

Monte Carlo Simulation

With our null hypothesis, the rotated occurrences are a random sample without replacement from the 241 occurrences of the 45 signs, taking into account for each sign the exact number of its occurrences. The probability distribution of the number of rotated signs can be computed with any desired precision using Monte Carlo with a sufficiently large number of replications, where each replication consists of a random sample.

This procedure was repeated with a different random “seed”, in order to estimate the accuracy of the estimated distribution. Thirty million random replications were used to obtain a sufficient accuracy.

The program has been written in both the R programming language and the Octave/Matlab programming language, which decreases the possibility of programming errors. Also, one of the programs has been tested with a toy version, with very few signs and few rotated occurrences. For this toy version the exact distribution can be obtained analytically and compared with the Monte Carlo result.

Table 1 shows the resulting probability distributions. The accuracy of the computations is such that with this rounding the result does not depend on the random seed or on the computer program. The “+0” cells indicate that with at least one of the various combinations of the random seed and the computer program, none of the 30 million simulations filled this cell, while of course the probabilities associated with these cells are not *exactly* zero.

The first line of the table shows the probability distribution without sign 27. The probability of only two rotated signs (or less) is extremely small: approximately 0.0003 percent. This is well below the usual statistical threshold of 5 percent.

The second line of the table gives the result including sign 27, with 10 rotations. The probability of only three rotated signs (or less) is approximately 0.0007 percent. This is also extremely small by any standard, although much larger than the probability of 10^{-8} obtained for this case from the Occupancy model above.

Hence the result of Table 1 is overwhelmingly against the null hypothesis of coincidence.

Table 1. The probability distribution of the number of rotated signs, under the null hypothesis

Number of rotated occurrences	Number of rotated signs										Total
	1	2	3	4	5	6	7	8	9	10	
	%										
8	+0	0.0003	0.03	0.6	6	24	43	27			100
10 (with sign 27)	+0	+0	0.0007	0.03	0.6	4	17	34	32	12	100

By mistake or on purpose?

This still leaves the question: are the rotations made by mistake or on purpose? (Other than the purpose of saving space, which was excluded above.) Assuming that the concentration among only two or three signs is not coincidental, as discussed above, the mistake or purpose must be related to these particular signs. This can be due to (a) the shape of the sign itself or (b) the “seal” with which a sign is stamped into the clay.

As to the former, the signs 29 and 31 are not hard to position correctly. Compare them with signs 33 (Tunny) or 37 (Papyrus), which might more easily be placed upside down by mistake.

This leaves the seals. Let us assume for the moment that for some reason the seal of 29 and the seal of 31 differ from all other seals. Considering the apparent care with which the Disc is made, it is unlikely that exceptional seals have led to careless use of them, continuing after several errors have been made.

Conclusion

It is – practically speaking – impossible that the rotations (other than to save space) on the Phaistos Disc are randomly distributed over the signs. Hence it is worthwhile to consider the hypothesis that these rotations are on purpose (other than to save space). This would shed new light on the nature of this artifact.

Of course, it is left to the archaeological profession to investigate this further. I limit myself to the suggestion that it may give some support to Whittaker [5], page 32, who suggests that the inscription might be fake writing, made by an illiterate person who imitates writing.

Acknowledgement

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