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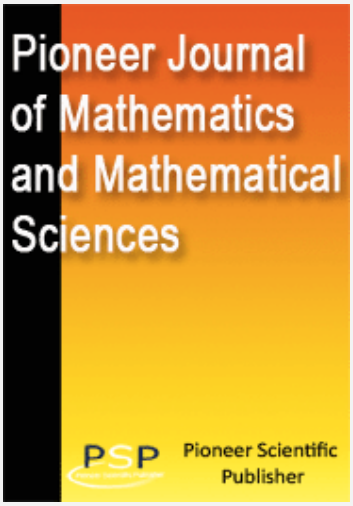

CONTEMPLATING SOME INVARIANTS OF THE JACO GRAPH, $J_n(1)$, $n \in \mathbb{N}$

Johan Kok and Susanth C

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Abstract

Kok et al. [J. Kok, P. Fisher, B. Wilkens, M. Mabula and V. Mukungunugwa, Characteristics of finite Jaco graphs, $J_n(1)$, $n \in \mathbb{N}$, arXiv: 1404.0484v1 [math.CO], 2 April 2014] introduced Jaco Graphs (order 1). In this essay, we present a recursive formula to determine the independence number $\alpha(J_n(1)) = |\mathbb{I}|$ with, $\mathbb{I} = \{v_{i,j} \,|\, v_1 = v_{1,1} \in \mathbb{I}$ and $v_i = v_{i,j} = v_{(d^+(v_{m,(j-1)})+m+1)}\}$. We also prove that for the Jaco Graph, $J_n(1)$, $n \in \mathbb{N}$ with the prime Jaconian vertex v_i the chromatic number, $\chi(J_n(1))$ is given by

$$\chi(J_n(1)) \begin{cases} = (n-i)+1, & \text{if and only if the edge } v_i v_n \text{ exists,} \\ = n-i, & \text{otherwise.} \end{cases}$$

We further our exploration in respect of domination numbers, bondage numbers and declare the concept of the Murtage number[1] of a simple connected graph G, denoted $m(G)$. We conclude by proving that for any Jaco Graph $J_n(1)$, $n \in \mathbb{N}$ we have that $0 \leq m(J_n(1)) \leq 3$.

Keywords and phrases: Jaco graph, Hope graph, independence number, covering number, chromatic number, domination number, bondage number, Murtage number, d_{om}-sequence, compact γ-set, Murtage partition.

[1] In honour of U. S. R. Murty, co-author of [2].