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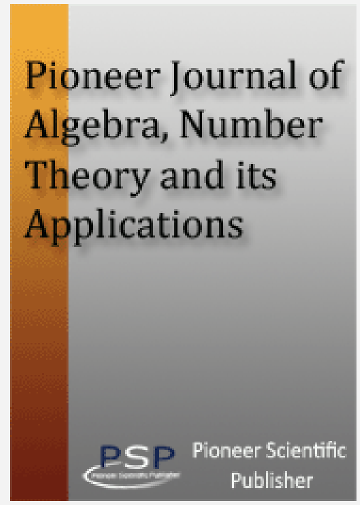


# PRIME POWERS AND GENERALIZED BENFORD LAW

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**Abstract**

It is known that the first digits of prime numbers follow a generalized Benford law (GB) with size-dependent exponent that converges asymptotically to the uniform distribution. Based on two different statistics, we show the existence of size-dependent exponents that outperform in precision the optimal size-dependent exponent in Luque and Lacasa [B. Luque and L. Lacasa, The first-digit frequencies of prime numbers and Riemann zeta zeros, Proc. Royal Soc. A 465 (2009), 2197-2216] uniformly over some finite ranges. This result also holds for prime squares. Extending the approach to prime powers, different rates of convergence of the size-dependent GB's to a GB with inverse power exponent are determined and compared. Furthermore, we introduce a criterion of counting compatibility, which indicates whether or not a given size-dependent GB that belongs to the first digits of some integer sequence is compatible with the asymptotic counting function of this sequence if it exists. We show the existence of a one-parametric size-dependent GB for the sequence of prime powers that is counting compatible with the prime number theorem and determine its optimal size dependence. Finally, based on the theory of distribution functions of integer sequences, it is proved that the first digits of prime powers converge asymptotically to a GB with inverse power exponent. In particular, asymptotically as the power goes to infinity the sequences of prime powers obey Benford's law.

**Keywords and phrases:** regular rings, principal ideals, Boolean algebra, idempotents, prime ideals, maximal ideals, open base, topological space, clopen sets, totally disconnected space, Hausdorff space, compact space.