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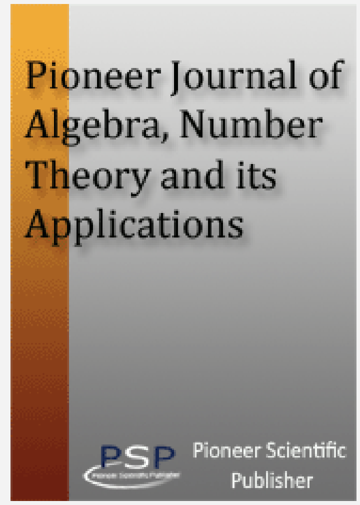

POSITIVITY OF HOCHSTER THETA

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Abstract

M. Hochster defines an invariant namely $\Theta(M, N)$ associated to two finitely generated module over a hyper-surface ring $R = P/f$, where $P = k\{x_0, \ldots, x_n\}$ or $k[X_0, \ldots, x_n]$, for k a field and f is a germ of holomorphic function or a polynomial, having isolated singularity at 0. This invariant can be lifted to the Grothendieck group $G_0(R)_{\mathbb{Q}}$ and is compatible with the chern character and cycle class map, according to the works of W. Moore, G. Piepmeyer, S. Spiroff and M. Walker. They prove that it is semi-definite when f is a homogeneous polynomial, using Hodge theory on Projective varieties. It is a conjecture that the same holds for general isolated singularity f. We give a proof of this conjecture using Hodge theory of isolated hyper-surface singularities when $k = \mathbb{C}$. We apply this result to give a positivity criteria for intersection multiplicity of proper intersections in the variety of f. Another proof of positivity for Θ via Mukai pairing is given, together with a formula describing Θ in terms of Mukai product on Hochschild Homology.

Keywords and phrases: matrix factorization, Riemann-Hodge bilinear relations, residue pairing, cyclic homology.