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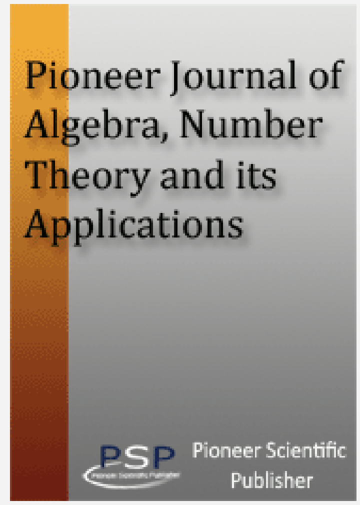


# A SIMPLE PROOF OF THE FRIENDLY NUMBERS PROBLEM VIA COMPLEX CALCULUS

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## Abstract

The notion of a friendly number (or amicable number) (see [1] or [5] or [6]) is based on the idea that a human friend is a kind of alter ego. Indeed, Pythagoras wrote (see [2] or [5] or [6]): A friend is the other *I*, such as are 220 and 284. These numbers have a special mathematical property: each is equal to the sum of the other's proper divisors (divisors other than the number itself). The proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110, and they sum to 284; the proper divisors of 284 are 1, 2, 4, 71, and 142, and they sum to 220. So {220; 284} is called a pair of friendly numbers [note {17296; 18416} is also a pair of friendly numbers (see [5] or [6]) and {12285; 14595} is a pair of friendly numbers too]. More precisely, we say that a number $a'$ is a friendly number or amicable number, if there exists a number $a'' \neq a'$ such that $\{a', a''\}$ is a pair of *friendly* numbers [Example: 220, 284, 12285, 14595, 17296 and 18416 are each friendly numbers]. It is immediate that a *friendly number* is a *composite number* (we recall that a composite number is a non prime number), and original characterizations of composite numbers via divisibility are given in [1] and [3] and [4]. Friendly numbers are known for some integers $> 18416$ and the friendly numbers problem states that there are infinitely many friendly numbers. In this paper, we give a simple proof of the friendly numbers problem by reducing this problem into a trivial equation of three unknowns and by using elementary combinatorics coupled with elementary arithmetic calculus, trivial complex calculus and elementary computation via the reasoning by reduction to absurd using friendly numbers. Moreover, our paper clearly shows that divisibility helps to characterize composite numbers as we did in [1] and [3] and [4], and trivial complex calculus coupled with elementary computation and elementary arithmetic calculus help to give a simple proof of the friendly numbers problem.

**Keywords and phrases:** prime number, number theory, distribution of prime numbers, the law of prime numbers, the Riemann hypothesis, the Riemann zeta function, the Mertens function, the Gamma function.

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