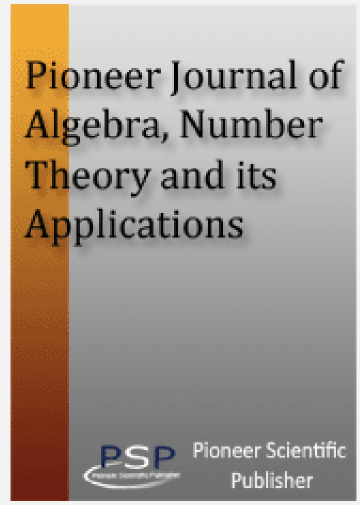
Pioneer Journal of Algebra, Number Theory and its Applications

Volume 11, Issues 1-2, Pages 1-19 (March & June 2016)

ISSN: 2231-1831



ON THE NUMBER OF PRIMITIVE PYTHAGOREAN SEXTUPLES

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Received March 23, 2016

Abstract

We express the number of distinct primitive Pythagorean sextuples in terms of thirteen components. These are the total number of primitive representations of a square as a sum of five squares (counting zeros, permutations and sign changes), the number of distinct primitive Pythagorean quadruples and quintuples, the number of distinct primitive squares quaternary representations $w^2 + x^2 + y^2 + 2z^2 = t^2$, the number of distinct primitive squares representations by the three ternary quadratic forms $x^2 + by^2 + cz^2$, $(b, c) \in \{(1, 3), (2, 2), (1, 2)\}$, as well as the number of distinct primitive squares representations by the six binary quadratic forms

$$ax^2 + by^2, \quad (a, b) \in \{(1, 4), (2, 3), (1, 3), (2, 2), (1, 2), (1, 1)\}.$$

Explicit formulas for each component are provided, and the obtained summary counting function is illustrated numerically. In particular, it is shown that the Pythagorean sextuple Diophantine equation $v^2 + w^2 + x^2 + y^2 + z^2 = t^2$ has primitive solutions without zeros for all $t \geq 4$. .

Keywords and phrases: diophantine equation, sum of squares, quaternary quadratic form, ternary quadratic form, arithmetic function, twisted Euler function.