ON BRUCK LOOP AND ITS PARASTROPHS
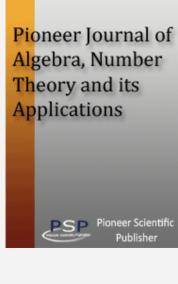
S. O. Ajala, J. O. Aderohunmu, S. O. Ogunrinade and A. O. Oyem

Abstract

Consider a quasigroup $(H, \cdot)$ as given in [Hala O. Pflugfelder, Quasigroups and Loops: Introduction, Sigma Series in Pure Mathematics, 7, Heldermann, Verlag, Berlin, 1990], Pflugfelder showed that there are five associated quasigroups which are called parastrophs. In this paper, we show that if $(H, \cdot)$ is a Bruck loop and it satisfies the automorphic inverse property (AIP) $(a \cdot b)^{-1} = a^{-1} \cdot b^{-1}$, then two of the five parastrophs are also Bruck loops. Bruck loops have applications in special relativity. If in addition to the above, the Bruck loop is commutative, then all the five parastophs are also Bruck loops.