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# CROSS PRODUCTS AND GLEASON FRAMES

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**Abstract**

In a physical energy explosion, after time is generated, an octonian Fano model allows a 7-dimensional projective extended complex spacetime $(\mathbb{C}^3, S^1)$ for energy carrying systems[1] $P$. The three quaternionic Pauli matrices show up in the Fano figure as seven cross products *GF* where the Pauli spin is only one of them. The new dimensions are for Higgs mass, frequency and a rolled light coordinate $S^1 \sim U(1)$. The rolled coordinate is for periodic functions (in complex polar coordinates) and shows up as fibre of the two $S^3$, $S^5$ fibre bundles, belonging to the geometry of the weak and strong nuclear interactions. Essential tools for this model are beside the *GF* as metrical Gleason frames, the norming of $S^5$ by $S^1$ to the complex projective space $\mathbb{CP}^2 \sim \mathbb{C}^2 \cup S^2$ for $P$ with bounding 2-sphere and, as its symmetry, Moebius transformations which allow pole singularities for spacetime.

[1] $S^n \subset \mathbb{R}^{n+1}$ are unit spheres with radius $r = 1$.

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In [3] the construction of Gleason operators $T$ can be read. They extend as complex 3-dimensional operators the quantum mechanical Pauli spin operator $S = (S_x, S_y, S_z)$. The construction of $T$ requires a frame, - the choice of an origin O, a localized 3-dimensional real subspace $U \subset \mathbb{C}^3$, where the $T$-induced metric is real, and choosing a real orthogonal triple of vectors with weights $(0 \leq m \in \mathbb{R})$ attached. The vectors $\rightarrow$ have their initial points in O, the weights can be spin vector length. They alternatively have their initial points $m_j$ on a 2-dimensional sphere in $U$ with center O such that $\mathrm{O}m_j$ is the orthogonal triple for $T$ and the $m_j$ are its weights. They apply as well to coordinates as to the differentials in the tangent bundle of an energy carrying system. The $\rightarrow$ vectors get a field interpretation which means that they can represent (with mass for instance as initial scalar for the Gleason operator) the energy qualities of an angular or linear momentum or a potentials direction. Mass can be replaced, for instance by induction, setting a north pole for a magnetic momentum vector, pointing to its south pole.

The best known example is the above named $T_{s,L}$ for spin which sets the length of the spin coordinates and generates the Euclidean metric on space $\mathbb{R}^3$. On the differentials the operator generates the Laplacian $\nabla^2$ for differential equations. Using the transformation rules for spherical to linear Euclidean coordinates, the operator also allows radial and linear $(d/da)$ differentiations or $da$, $a = r, x, y, z$, integrations.

In the internet literature, available under the search 7-*dimensional cross product* or *octonions* a list of seven such cross products, all given by reflections like the Pauli spin matrices, can be found. $T_{s,L}$ is one of them. Before we propose a list of the other 6 Gleason operators, we mention the generation of other useful measures, obtained from the Hopf map $h : S^3 \rightarrow S^2$ for the $\mathrm{SU}(2)$ spin symmetry of the weak interaction:

The first Hopf coordinate uses the Pauli matrix $\sigma_1$ and applies to complex coordinates $z_1$, $z_2$ generating the complex dot product

$$z_1 \cdot z_2 = |z_1||z_2|\cos\theta,$$

where $0 \leq \theta \leq \pi$ is the angle between $z_1$ and $z_2$;

The second Hopf coordinate uses the Pauli matrix $\sigma_2$ and generates the complex cross product $z_3 = z_1 \times z_2 = |z_1||z_2|\sin\theta$; if $z_3$ is drawn orthogonal to the complex $z_1 z_2$-plane, this is used to blow spacetime $z_1 = z + ict$, $z_2 = x + iy$ up from complex 2 to complex 3 dimensions $\mathbb{C}^3$; $\sigma_2$ is also used for the sign of the electrical charge of weak bosons $W^{\pm}$ in their coordinate presentation $S_x \pm iS_y$; in the tangent bundle the differentials $d\varphi$, $(d/d\varphi)$ for integrations and differentiation are set;

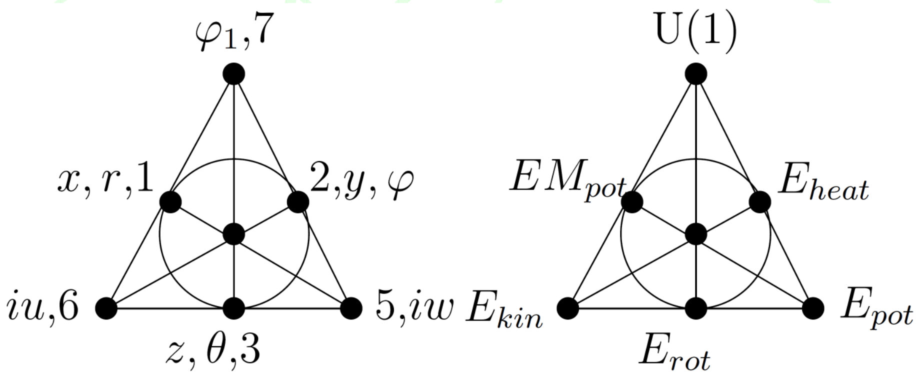


**Figure 1.** Projective Fano model.

The third Hopf coordinate uses the diagonal Pauli matrix $\sigma_3 = \mathrm{diag}[1, -1]$; it generates a quadric $[z_1, z_2]\sigma_3[\overline{z_1}, \overline{z_2}]^{tr} = |z_1|^2 - |z_2|^2$; in case it is applied to polar coordinates $z_2 = re^{i\varphi} = r(\cos\varphi + i\sin\varphi)$ and $z_1 = ict$, the quadric of the Minkowski light cone[2] is obtained $c^2t^2 - r^2$ by setting $c^2t^2 - r^2 = 0$; if $z \neq 0$, write the quadric as $(z^2 + c^2t^2) - (x^2 + y^2)$ which gives projective a one-sheeted hyperboloid by setting $(z^2 + c^2t^2) - (x^2 + y^2) = 0$; its projective closure,

[2]For ordinary (not dark) matter with mass, its momentum $p$ is a space-like vector, pointing in the outer part of the light cone. This means, its speed measured as $v = (\Delta x)/(\Delta t)$ satisfies $v \leq c$.

including the points at infinity, is the torus, a geometry[3] for leptons; in the tangent bundle the differentials $d\theta,\ (d/d\theta)$ for integrations and differentiation are set;

Reflections squared give the identity operator *id*; when projective applied for quadrics (as for the $\sigma_j$), it gives as fourth Pauli matrix the complex norm $[z, 1](id)[\bar{z}, 1]^{tr} = z\bar{z} = |z|^2$ for measuring length.

Hence the Pauli matrices introduce not only the Euclidean metric and the Laplacian, but also the important measures for complex numbers, the blow up of spacetime to $\mathbb{C}^3$ and special relativistic and leptonic geometrical quadrics.

This setting can be extended to other Gleason operators[4] which in this article is mostly left as open problems for (experimental) research(ers).

Since the energy space, used in [3] for unifying gravity and mass with the standard model of physics, is 7-dimensional we can accommodate the 7 coordinates *m* on a Fano figure with lines as the seven cross products in [1] such that to the seven coordinate-points *m* three lines as cross products are available. In the figures, the barycentrical midpoint of the triangle is named *ict*, 4, $E_{magn}$. The numbers are for the 7 octonian reflections.

As proposal, the octonian seven cross products are then for Gleason operators, integrating in a nucleon (through the energies and gluon exchange between paired quarks) mathematically the following list of energies:

1, $EM_{pot}$, electromagnetic potential, electrical charge, radius, line 123, with Moebius transformations MT translations.

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[3]Draw in the $(x, y)$-plane about 0 a circle $C_1$ of radius *r*; draw a second circle $C_2$ of radius *r* in the $(y, z)$-plane with center $(0, a, 0)$, $a > r$ and let $C_2$ rotate about the *z*-axis. The point 0 can be considered as an electrical, neutral or magnetic charged pole of the lepton. In the Hopf fibre bundle with fibres $S^1$, the circle $C_1$ is a fibre, for instance as $h^{-1}(\infty)$, $\infty \in S^2$.

[4]In [3] a Gleason operator *rgb*, called graviton, is set for the neutral color charge of hadrons. It is not in the Fano *T* list.

2, $E_{heat}$, heat equations, phonons, stochastic energy-momentum transfer, 246, flows, possibly with one or two poles as sink and source or as dipoles $0,\ \infty \in S^2$, 2 polar MTs $(z-a)/(z-b),\ a, b \in S^2,\ a \neq b$ or normed

$$1/(z-b),\ b \neq \infty \text{ or } 1/z,$$

with one pole[5].

3, $E_{rot}$, angular momentum, strong interaction 6-cyle of [3], gluon whirls, 347, MT rotations

$$A = (a_{ij}),\ a_{ii} = \cos\varphi,\ a_{12} = \sin\varphi = -a_{21}.$$

4, $E_{magn}$, magnetic momentum[6], 145, dilations $\mathrm{diag}[a, b],\ 0 \neq a,\ b \in \mathbb{R}$, as special relativistic stretching and squeezing, the symmetry $\{id, M\}$ with the scaled MT $M = (m_{ij})$ with trigonometric coordinates for a variable speed $v/c$, in the tangent bundle $dt$, $(d/dt)$ for time differentiation and integration

5, $E_{pot}$, gravitational potential, particles with barycenters, 257, complex inversion at the Schwarzschild radius $C$ as circle

$$|\,0P\,|\,|\,0P'\,| = R_S^2,\ 0, P,\ P' \in S^2;$$

lemniscate with two focii for the (electrical and color charge) poles of a quark as inverse image of a hyperbola, the MT modular group symmetry.

6, $E_{kin}$, momentum $p = mv$, 365, linear or angular speeds for energy systems $P$, orbits (for instance as conic sections for planets[7]), the two cosmic speeds for $P$ with mass, the SI 6-cycle with the MT symmetry $D_3$ of a nucleons quark triangle.

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[5] Source and sink free flows in $\mathbb{C}$ are obtained from analytic functions $\Omega$, poles $a$, $b$ can be included for such potentials $\Omega$.

[6] Observe also the EM charge induced changes $\pm$ between the aligned magnetic momentum and spin.

[7] See for instance [3] for general relativity correcting Kepler when angular $E_{rot}$ frequency applies.

For the brezel geometry of quarks, [6] can be consulted. Under some natural assumptions, the line, circle and lemniscate[8] are characterized as the only algebraic curves $\Gamma_{\mathbb{R}}$ in the projective complex space $\mathbb{CP}^2$.

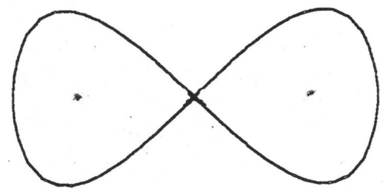

**Figure 2.** Lemniscate.

The coordinate 7 is rolled, as in the Kaluza-Klein theory, but here used for the U(1) circle symmetry of the electromagnetic force with photons as energy carriers, for (harmonic) wave equations, $\psi$-functions; for this, the 176 line is extended 4-dimensional by the *ict*, 4-coordinate. The 176 geometry is a circular cylinder *Z* for periodic functions like $e^{i\varphi_1}$ with a fundamental domain. For double periodic functions, use a rectangle and identify opposite sides to obtain a leptonic torus. $\varphi_1$ can in addition be used for a projector as in [8] or for bags, norming projectively the

[8] Nucleons are open systems. In the SI 6-cycle the maximal length of quark lemniscates and their distances are changed. Contractions (expansions) are by decreasing (increasing) temperature in the nucleon. After heat and gravitons are emitted from the nucleon, $EM_{pot}$ is integrating the nucleon (neutral or) +-charge as positron where the 3 quark spins are set (anti-) parallel and the nucleon spin aligned with its magnetic momentum is generated. The nucleons inner entropy decreases as well as its volume, increasing its angular momentum and decreasing its inertial mass momentum. In the following Laplacian integration for an inner harmonic wave as superposition for the blue colored quarks motion in the 6-cycle also energy is absorbed and the volume decreases until the GR potential $E_{pot}$ is integrated and gravitons are generated. The measures of the magnetic momentum of quarks suggest that now the number of measurable items in the nucleon is increasing, maybe their spins are not aligned, the positron is annihilated for the measurable $(+2/3)$, $(-1/3)$ EM-poles of quarks. This increases again inner entropy. After that in the 6-cycle the increased (via contraction) $E_{rot}$ is integrated, inertial mass momentum of the nucleon increases again by a generated graviton field. Increased is also the length of the quarks lemniscates and their distances. Only approximating models can be suggested for this open system, no exact computation as that of a Schrödinger wave.

fibres $S^1$ of the fibre bundle[9] $S^5 \subset \mathbb{C}^3$ to $\mathbb{CP}^2$ for local spacetime coordinates of particles bag as a bounding Riemannian sphere $S^2$ of radius *r*. The symmetries of $S^2$ are the complex Moebius transformations MT[10]. In octonians, 176 applies as oriented Gleason frame to isospin with electrical charge *Q* and isospin $I_z$ $\pm$ oriented $(x, z)$ or $(-x, -z)$ rotations about the $(iy)$-axis *Y*. Changing from $+$ to – means a $180^0$ rotation[11] about *Y*.

Since the Fano projective subspace duality $0 \leftrightarrow 3$ requires 4 dimensions, the 3-dimensional Gleason subspaces can be extended higher-dimensional as follows.

Pauli spin for space is extended to spacetime 1234, 4, the *ict*-coordinate[12]. Its Minkowski metric[13] for the weak interaction is listed as a diagonal matrix

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[9] $S^5$ is in WIGRIS presented dynamical as $D_3$ symmetry with 3 reflections and 2 orientations of the nucleon triangle. There are two poles $E_{rot,\,pot}$ for integrations; $E_{kin}$ is rotated clockwise cw (counterclockwise mpo) with $E_{pot}$ $(E_{rot})$ in the SI 6-cycle (see below for more details). Recall, that the toroidal geometry of SI is $S^3 \times S^5$.

[10] In [3] coordinates axes (see Figure 4 and its added comments) of a Gleason operator 356 in the Fano figure have on a nucleons $S^2$ MT poles of force vectors attached; three vectors are as the for the SI 6-cycle on the negative ends of the axes while on the positive ends of the axes are the other 3 energy vectors listed to provide the Heisenberg uncertainties HU, -paired in coordinates as $(x, iw)$, $(\varphi, \theta)$, $(ict, iu)$. The HU generate grids with lower bounds for measurements by the laws $\lambda \cdot p = h$ (position-momentum or angle-angular momentum) and for the last one $E = hf$, *f* as inverted time interval, where also Planck numbers can be used with $t_P \cdot E_P = h$.

[11] The use of an *iy*-coordinate means to express polar coordinates $re^{i\varphi_1}$ in terms of complex linear $x + iy$ coordinates.

[12] Observable $S^3$ can be mesons, having a short lifetime. The two lemniscate-brezel surfaces rotate against one another before a Heegard decomposition occurs and the quarks decay further according to Feynman diagrams and Gleason.

[13] SR uses for coordinate scalings an MT $\delta = (d_{ij})$ of order 2 with $d_{11} = -1$, $d_{21} = 0$, $d_{j2} = 1$.

$\mathrm{diag}[1, 1, 1, -1]$ with the Sylvester notation for quadrics, norming the coefficients to $\pm 1$. The Einstein affine minimal model is contained in the Fano figure as 1234 spacetime. It can be used for a weak 4-cycle, possibly extended to 5 dimensions by the lights U(1)[14],[15]. Generated are the 3 weak bosons, the photon, spin in two versions, for neutral leptons as helicity, for electrical charged ones as conic rotating whirl with magnetic momentum aligned.

The strong interaction SI 365 has a 5-dimensional spin and according to the table below a $\mathbb{C}^3$ extension 123465 with $\mathrm{diag}[1, 1, 1, -1, -1, -1]$. An energy plane $(iu, iw)$ for frequency and mass extends spacetime to the operator space $\mathbb{C}^3$ for complex Gleason operators, acting also on tangent bundles. Rotations as matrices of order 3 are for the SI 6-cycle, using the non-commutative MT symmetry group $D_3$ of order 6, belonging to the quark triangle in a nucleon. Integrated are: a bag radius

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[14] In earlier publications, the Hopf map was used to generate the EM charged leptons geometry in spacetime: Put the central circular core $x^2 + y^2 = const.$ as image $st_3 \circ h^{-1}(\infty)$, $\infty \in S^2$ of the stereographic north pole with $\infty \notin st_2(S^2)$. For the complex planes $z_1 = z + ict$, $z_2 = x + iy$ set the vectors orthogonal which means $z_1 \cdot z_2 = 0$. Since $z_1 = 0$ is the origin O in the *xy*-plane, this means that the *z*-axis is orthogonal to the *xy*-plane and acts as rotation axis for the tori $st_3 \circ h^{-1}(C)$, $C$ latitude circles in $S^2$. Since the $S^2$, $st_2(S^2)$ coordinates are $z = z_1/z_2$ under the Hopf map, an EM charge $q$ pole rotation on some $C$ is described in polar coordinates as $z = re^{i\varphi}$, where the mass-charged core is kept fixed and the tori as $h^{-1}(C)$ are rotating cw or mpo about the *z*-axis. The $S^3$ rotations (with fixed core) are given by a special relativistic angle of the spin aligned with magnetic momentum $\mu$ of $45^0$ by using as matrix $A = (1/\sqrt{2}) \cdot (id + i\sigma_2)$. In this geometry, $ict$ in form of inverse rotational frequency $\omega = (2\pi)/\Delta t$ is not taken orthogonal to space coordinates. By induction, the tori and spin rotations have the same $\omega$, $\mu$ has its north pole in O, spin points in direction of the positive *z*-axis and the south pole of $\mu$ in $\pm$-direction for the $\pm$-charged leptons rotation. $\mu$ has to be orthogonal to the leaning circle $h^{-1}(q) \subset h^{-1}(C)$.

[15] The above $+45^0$ rotation is extended (by using the magnetic group (see Figure 3) with $i \cdot A$) to $-45^0$ rotations. This group is also for the WI 4-cycle (see [3] using the matrix $i\sigma_2$).

$r = \int dr$ and $EM_{pot} = F_{EM}$ to the EM potential $-a \cdot (q/r) = \int F_{EM}\, dr$, a nucleons inner heat flow $f$ between two concentric circles as differential equation $d(rf)/dr^2 = 0$, angular momentum $M$ to $L = r \times p = \int Mdt$ for rotations, a wave equation for oscillations, magnetic flow $\Phi$ using voltage $U$ as induction $N \cdot \Delta\Phi = \int U(t)\,dt$ ($N$ winding number of a coil), Newtonian force $F_N$ to momentum $p = mv = \int F_N\, dt$, gravitational force $F_G$ to its potential $-(\gamma_G m)/r = \int F_G dr$.

In the internet under YouTube the action of MTs are demonstrated (translations, rotations, inversion, stretching and sqeezing[16]). The stereographic projection $st_2$; $S^2 - \{\infty\} \to \mathbb{C}$ is used.

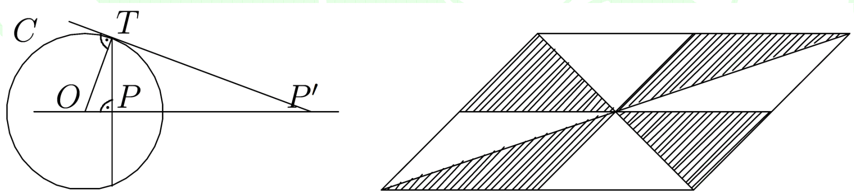


**Figure 3.** Inversion, magnetic group.

I add for[17]. periodic functions the use of strips $|\, x \,| \le 1$ or rectangles $|\, x \,|, |\, y \,| \le 1$ for fundamental domains, using the inverse map $st_2^{-1}$. In the list for 2-dimensional compact, connected, orientable manifolds, $S^2$ has genus 0, the last mentioned torus has genus 1 with the standard edge notation $aba^{-1}b^{-1}$ and the quark brezel has genus 2 with edges $aba^{-1}b^{-1}cdc^{-1}d^{-1}$. These genus 1, 2 surfaces arise

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[16] The below mentioned $R_S$ matrix $\beta$ for $60^0$ angle turns has as square root the matrix for spiralic $30^0$ angles $\eta = (1/\sqrt{3})(c_{ij})$, $c_{11} = 2 = 2c_{2j} = -2c_{12}$.

[17] For the postulated spin 2 of gravitons I mention from old publications that the spin lengths occur as degenerate orbit of the $D_3$ group. Signed spin $-1/2$ is for fermions, spin 1 for weak bosons and gluons in a nucleon, hence spin 2 and the subgroup of $D_3$ of order 3 can have a particle presentation which means this particle is the searched graviton. A periodic function is not used.

from the weak $S^3$ Heegard splitting into two 3-dimensional handlebodies[18].

The use of octonians as number system is otherwise not recommended for the WIGRIS model in [3]. The MTs are in addition to reflections: the cubic $\alpha \in D_3$ and a MT $\beta = (b_{ij})$, $b_{j1} = 1$, $b_{12} = -1$, $b_{22} = 0$ of order 6 for the general relativistic scaling[19] of the metric differentials in a tangent bundle of a local spacetime which belongs to central system (such as suns); β is responsible for setting 6 masses of the two fermionic (quark and lepton) series; applying also the conjugation operator C, the series have 12 members.

A combined list with coordinates and relevant datas are presented in the following table, columns belong together:

In the first line of the table are the spherical SI coordinates, possibly 7- (not 8-) dimensional extended with exponential/polar coordinates. In the second line are the linear Pauli/Euclidean coordinates, in the third line a distribution of color charges to the SI coordinates. The fourth (fifth) line contains the $D_3(\mathrm{SU}(2)/\mathrm{Pauil})$ MTs as cross ratios. Their matrix names are in the sixth line, together with the Einstein matrices. The following line is a numbering for a strong 6-fold integration series (not the Fano figures numbers which are for octonians). The next line contains the Planck numbers. Energy vectors are in the second to last line and the last line contains natural constants and three more operators, $C$ (conjugation for quantum numbers),

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[18] Fundamental domains FD arise also in other forms: the magnetic group has 8 FD, dividing a quadrangle by its 2 diagonals and 2 intervals connecting the midpoints of its sides. A coloring of the triangulated surface of a cube with the color charges red, green blue is found in [3].

[19] $b_{12}$ is scaled by the Schwarzschildradius $R_S$ and $b_{j1} = r$ is a variable radius. The nonlinear scaling is in the tangent bundle for differentials, used in the Minkowski metric for systems with mass. β used as $60^0$ rotations $(k\pi)/6$, $k = 0, 1, ..., 5$, generates also the 6 EM charges. For 2-state systems in physics, the 6 permutations of the numbers 123, realized in the SI 6-cycle, can be used where the $360^0$ rotation is associated with the matrix –*id* and only the $720^0$ rotation gives *id*. One or 3 inversions in the permutation are for right-hand screws in space, the other ones with an even number of inversions in the permutation for left-hand screws in space.

$T$ (time reversal) and $P$ (space parity) of physics. The CPT preservation is in $D_3$ matrix multiplication $\alpha\sigma_1 \cdot \sigma_1 \cdot \alpha^2 = id$.

| | | | | | |
|---|---|---|---|---|---|
| $r$ or $re^{i\varphi_1}$ | $\varphi$ | $\theta$ | $ict$ | $iu$ | $iw$ |
| $x \in \mathbb{R}$ | $iy \in i\mathbb{R}$ | $z \in \mathbb{R}$ | | | |
| $r$ | $g$ | $\overline{g}$ | $\overline{b}$ | $b$ | $\overline{r}$ |
| $z$ | $\frac{z}{-z-1}$ | $\frac{-z-1}{z}$ | $(-z-1)$ | $\frac{1}{-z-1}$ | $\frac{1}{z}$ |
| $\frac{1}{z}$ | $-\frac{1}{z}$ | $-z$ | $z$ | | |
| $id; \sigma_1$ | $\alpha\sigma_1; \sigma_2$ | $\alpha^2; \sigma_3$ | $\alpha^2\sigma_1; id$ | $\alpha; \delta$ | $\sigma_1; id; \beta$ |
| 1 | 6 | 4 | 2 | 5 | 3 |
| length $\lambda_P$ | temp. $T_P$ | dens. $\rho_P$ | time $t_P$ | ener. $E_P$ | mass $m_P$ |
| $EM_{pot}$ | $E_{heat}$ | $E_{rot}$ | $E_{magn}$ | $E_{kin}$ | $E_{pot}$ |
| $c, e_0, \epsilon_0$ | $k$,C | $N_A$,T | $\mu_0$ | $h$ | $\gamma_G, R_S$,P |

Some parts from [5] related to this table are not reprinted in [3]. Gleason operator frames in the 1235 space (pseudometrics $\mathrm{diag}[1, \pm 1, 1, -1]$) for the $ud$-decays through $W^{\pm}$ bosons $u \to d + e^+ + \nu_e$, $d \to u + e^- + \overline{\nu_e}$ are added (the two brezel lemniscate poles are on $iw$ for color charges and on $S_x \pm iS_y$ for their electrical charge):

Quarks color charge, mass carrying vectorial whirls use the $iw$ coordinate as one pole of their lemniscate and the gluon $b\overline{r}$; their EM poles are using oriented $S_x \pm iS_y$ coordinates in the $xy$-plane and the gluons $r\overline{b}$ $g\overline{r}$; the neutral leptons emitted in these quark decays alignes $S_z$ as spin with linear momentum $iu$ for helicity, the gluons $r\overline{g}$, $b\overline{g}$; for electrical charged leptons, spin is aligned with magnetic momentum, the gluon $g\overline{b}$ as conic whirl, rotating about the neutral $S_z$-axis.

This $ud$-decay belongs to the SI 6-cycle. In the following table, lines belong together. In the columns, the first one contains the energies, the second one the GR size of the quark triangle as *sm*, *mi*, *la* (small, middle, large size), the third one the right or left location of the $p^+$, $n$ nucleons in a deuteron with the spin $z$-axis in between (pointing upwards), in the fourth column the list of the 6 quarks is shifted by one item always to the left for the SI cycle. A list of gluons, possibly used for integrations is in the last column. As seen, their are two changes of the

$p^+$, $n$ locations. For the small triangle, an inner GR diffusion differential equation is solved which generates gravitons. Also phonons are generated by $E_{rot,kin}$ integrations. Gravitons and heat (phonons) are emitted from the nucleon for the large size triangle in its environment. Reversing this process from the large to the small size triangle they are absorbed for those integrations.

| | | | | |
|---|---|---|---|---|
| $E_{pot}$ | *sm* | $p^+$, $n$ | *uuddud* | $b\bar{r}$ |
| $E_{rot}$ | *sm* | $n$, $p^+$ | *uddudu* | $r\bar{g}$ |
| $E_{kin}$ | *mi* | $n$, $p^+$ | *dduduu* | $b\bar{g}$ |
| $E_{heat}$ | *la* | $n$, $p^+$ | *duduud* | $g\bar{r}$ |
| $EM_{pot}$ | *la* | $p^+$, $n$ | *uduudd* | $r\bar{b}$ |
| $E_{magn}$ | *mi* | $p^+$, $n$ | *duuddu* | $g\bar{b}$ |

Concerning an earlier footnote on the 356 Gleason operator of the Fano figure, the hedgehog shows that not only $S^3$ is observable as mesons or weak bosons of short lifetime, but also through different kinds of projections $S^5$ is observable as the HU for paired energy or coordinate vectors. For the pairs $(z_1 = x_1 + iy_1, z_2)$ of SI coordinates, drawn as vectors with initial points on $S^2$, (but also for electromagnetic equation $\Phi_0 \circ e_0 = (h/2),$) the two vectors involved are either aligned or put orthogonal by special relativistic angles 0 or $\pi/2$ between them. If $z_2$ is the vector attached at the negative end of such an axis, belonging to the SI 6-cycle, then the vector at the other end is either (1) $z_1$ or (2) $-i\sigma_1 z_1$. In case (1) $z_1 \times z_2 = 0$ implies that the vectors are aligned, as in the hedgehog, in case (2) the same vectors are set orthogonal by $(-i\sigma_1 z_1) \cdot z_2 = 0,$ spanning a rectangle in a plane. This area can be relativistic stretched and squeezed, keeping its area $A$ constant. The angles between the two vectors can be sheared to arbitrary non-zero angles. There are lower bounds for grids arising from the earlier mentioned equations such that $A$ has as a lower bound a suitable multiple of the Planck number $h$. If all aligned vectorial pairs in the hedgehog are set orthogonal, the $\mathbb{C}^3$ operator space coordinates of the above table are obtained.

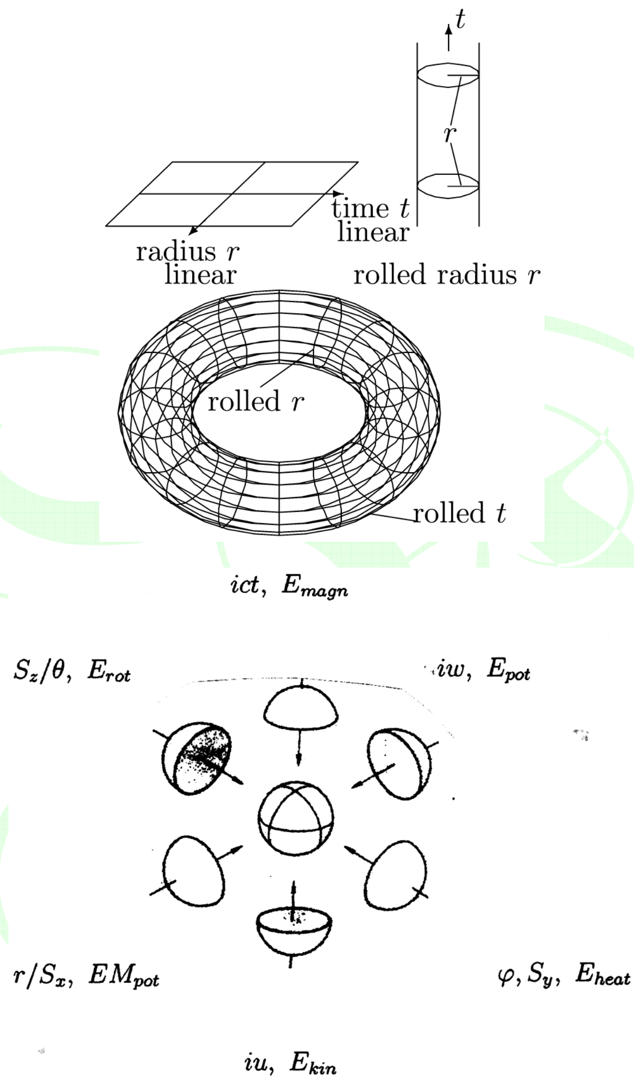


**Figure 4.** Rolled coordinates for periodic or double periodic functions, hedgehog.

In the $\mathbb{C}^3$ coordinate table I added for light a rolled seventh polar coordinate $z_w = re^{i\varphi_1}$ for its symmetry $U(1)$ and for periodic functions. Instead of the former HU rectangles, the magnetic vector aligned with time is postulated to be set orthogonal (or special relativistic sheared) to $\mathbb{C}^3$. The lights momentum shows helicity (like antineutrinos) and aligns with the positive $ict_w$ coordinate (not in $\mathbb{C}^3$, only parallel to $ict$) as expansion of its wave[20].

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[20] In a flat 2-dimensional reduced figure *P*, spacetime has as observable of the cylindrical helix line $z_w$ for lights energy $\psi$ wave, expanding in time, the projection of its cosine (or sinus) function. Only a projection *g* of its central axis on the cylinder touches *P*, interpreted as its spacetime expansion. In the old animation of 2000 it was shown how rolled coordinates as Lissajous figures can be generated: two frequencies, here postulated as electrical and magnetic vectors hit orthogonal. If they are in proportion 1 : 1, a circle is generated as U(1) (for the quark lemniscate the proportion is 1 : 2). This circle with two poles for its *E*, *M* induction is set orthogonal to spacetime and touches *P* in *g*. Using the Einstein formula $hf = mc^2$, a circular frequency is postulated as boundary of a small conic whirl *V*. It has its top O for a mass scalar on the $ict_w$ axis of the cylinder. The associated Gleason operator $T_w$ (with the center of U(1) as its origin) for light has the mass scalar at O, together for its vectorial frame with the *E*, *M* poles in U(1). - In experiments the cone as photon can show particle character. O moves with momentum on $ict_w$ with speed of light. The $T_w$ vector for the photon has the initial point O and its endpoint rotating on the cones U(1). The EM energy vector with initial point *E* is rotating as tangent to U(1) with its initial point on U(1). Observe, that in 7 dimensions $T_w$ can shift its sphere $S^2$ (in the cylinder) for its frame through the inverse stereographic map from a plane *Q* in $\mathbb{C}^3$ outside $\mathbb{C}^3$. The plane *Q* is spanned by *g* and $st_2$ projects U(1) from the diametrical opposite point of $g \cap U(1)$ as second coordinate in *Q*. This way $\Psi$ is observable.

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