Pioneer Journal of Theoretical and Applied Statistics
Volume 1, Issue 2, Pages 113-144 (April 2011)

ISSN: 2230-9837

TRANSFORMATION INVARIANT DISTRIBUTIONS AND FAMILIES: THE RECIPROCAL AND THRESHOLD PROPERTY

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Received January 22, 2011

Abstract

We consider the basic notions of strong and weak transformation invariance for random variables and their associated distributions in a parametric context. In the univariate case and some special multivariate situation, strong transformation invariant non-negative random variables with absolutely continuous distributions are characterized by functional relationships for their densities and transformed densities under monotone transformations. Reciprocal invariant distributions and families as well as scale and threshold invariant families are studied in greater detail. For the reciprocal transformation, we obtain slightly generalized versions of earlier results by Seshadri [Canadian Mathematical Bulletin 8(6) (1965), 819-824] and Saunders [Journal of the American Statistical Association 69(346) (1974), 533-539], and illustrate these at several examples. In particular, we display a new three-parameter reciprocal family, which is related to a generalized inverse Gaussian density, and whose maximum likelihood equations are explicitly determined. On the other side, it is shown that the generalized Pareto and a three-parameter extension of it satisfy the scale and threshold invariant properties. As an additional property, the threshold invariant parameters of these two families are expressed intrinsically in terms of the threshold invariant mean excess function. Finally, the difficult problem of the existence of strong invariant distributions is solved in a special case as corollary of a result in the theory of iterative functional equations. As illustration, it turns out that the only non-negative power invariant random variables, whose distributions are analytical at zero, are the reciprocal random variables. To motivate potential applications, we indicate some situations from insurance and finance where transformation invariance could be appropriate.

Keywords and phrases: strong invariance, weak invariance, functional equation, power transformation, generalized Pareto, Benktander distributions, Bowers distribution.