Pioneer Journal of Theoretical and Applied Statistics
Volume 11, Numbers 1-2, 2016, Pages 15-23
This paper is available online at http://www.pspchv.com/content_PJTAS.html

BI-LEVEL PROGRAMMING WITH DISCOUNT FOR DISTRIBUTION PROBLEM

Dr. ESSAM FAWZY and Dr. MAHMOUD EISSA

Department of Mathematics and Statistics
Faculty of Commerce
Ain Shams University
Egypt

Abstract

The bi-level programming for newsboy problem recognizes the manufacturer and multiple retailers as two separate entities and studies the interactive decisions between manufacturer and multi-retailers. In this paper, some models with different discount are presented, which are used to fix the wholesale price or discount so that the manufacturer can get the maximum profit. To solve these models, an efficient algorithm based on genetic algorithm and simulation is provided and some numerical examples are given.

1. Introduction

In the real world, many products have a limited selling period such as newspaper and Christmas tree, therefore, in the last decade; the newsboy problem has been studied widely. In contrast to the classical newsboy problem which considers only the benefit of retailer or manufacturer, the bi-level newsboy problem

Received April 4, 2016; Re-revised May 25, 2016

2010 Mathematics Subject Classification: 97K80.

Keywords and phrases: newsboy problem. bi-level programming, discount methods, genetic algorithm.

considers not only the manufacturer but also multiple retailers, in which the manufacturer and retailers may have their own decision variables and objective functions, and the manufacture can only influence the reactions of retailers through his own decision variables, while the retailers have full authority to decide how to optimize their own objective functions in view of the decision of the manufacturer. In the papers of Lin [1] and Khouja [2], manufacturer-retail is assumed to be a single integrated entity, hence one considered only the decision of a single decision-maker, it belonged to a single level model. In fact, many manufactures rely on many independent retailers to sell their single period products. If wholesale price is lower, the quantity ordered by retailers is more and the profit of manufacturer is also more. In this situation, there will be multilevel decision makers including manufacturer and retailers. The bi-level newsboy problem considers the interactive decisions between the manufacturer and retailers. Most researchers such as Lau [3] and [4] on newsboy Problem, compared the normative optimal behavior in two-echelon multiplier distribution systems for a single-period product. In this paper, a further step towards reality is to give the wholesale price with different discount methods according to the quantity ordered by retailers. In this paper, we extend the newsboy problem to the case in which multiple independent retailers with all unit discount and incremental discount are considered. To solve these models, an efficient algorithm based on genetic algorithm and simulation is provided.

2. Overview of the Newsboy Problem

The classical newsboy problem is to determine the number y of newspapers to order in advanced from the manufacturer so as to maximize the expected profit of retailer. In order to illusion the problem clearly, we give the following definition:

A: unit retail price

C: unit wholesale price

B: unit return price

D: unit manufacture price

Y: order quantity

E: demand variable.

2.1. Classical newsboy problem

In single level problem, newsboy problem can be stated as follows: given *a*, *b*, *c*, *e*, find an order quantity *y* to maximize newsboy's profit. We can formulate the density function of newsboy's profit as follows,

$$f(y, \xi) = \begin{cases} (a-c)y, & \text{if } y \leq \xi, \\ (b-c)y + (a-b)\xi, & \text{if } y \geq \xi. \end{cases}$$

In practice, the demand *e* for newspaper is usually a stochastic variable, so is the profit function $f(y, \xi)$. We can employ expected value model to find the maximum mean profit of newsboy. So the single level model can be formulated as follows,

$\{\max E[f(y, \xi)]$

subject to:

$y \geq 0,$ intergers,

where

$$E[f(y, \xi)] = \int_0^y [(b-c)y + (a-b)r]d\Phi(r) + \int_y^{+\infty} (a-c)yd\Phi(r).$$

In which *E* denotes the expected value operator and Φ is the distribution function of demand ξ.

2.2. Bi-level newsboy problem

A mathematical programming problem is classified as a bi-level programming problem when one of the constraints of an optimization problem is an optimization problem. The bi-level programming problem arises in many contexts. The bi-level newsboy problem can be expressed as follows: the manufacturer whishes to determine an optimal wholesale price χ, and the retailers' responses to these controls by adjusting the newspaper order quantity γ so that can get their maximum expected profit, the manufacturer then seeks to maximize his own expected profit function of both χ and γ, where some constraints may be imposed upon χ and γ.

In this problem, a lower-level decision maker only reports the quantity which he will order according to the wholesale price. If manufacturer gives a lower wholesale price, the more products will be ordered by the newsboys. The model is formulated as follows:

$\{\max(\chi - d)\, y$

χ

subject to:

$\chi \succ d$

where each y solves

$\max E[f(x,\ y,\ \xi)]$

y

subject to:

$y \geq 0,$ intergers,

where d is the unit cost price and y means the quantity ordered by the newsboy.

3. Bi-leveled Models with Different Discounts

The above discussion starts by offering the different retailers the same wholesale price without considering the discount. Now, assume to be based on competitive considerations, the manufacturer gives the different wholesale price according to the product quantity to improve the quantity ordered by retailers. We consider multiple retailers and the whole price with discount, the model can be formulated as follows,

$\{\max F(x,\ y_1,\ y_2,\ \ldots,\ y_n)$

x

subject to:

$x \succ 0$

where each y_j $(j = 1, 2, \ldots, n)$ solves

$\max E[f_j(x,\ y_j,\ \xi_j)]$

y_j

subject to:

$y_j \succ 0$, integers.

In this model, y_j, ξ_j mean the quantity and the demand of the *j*th newsboy.

In market, the manufacturer may follow different discount, if the manufacturer gives the retailers whole price with all-unit discount, the objective functions can be formulated as follows,

$$F(x, y_1, y_2, ..., y_n) = \sum_{j=1}^{n} [(g(x, y_j) - d) \times y_j],$$

and

$$f_j(x, y_j, \xi_j) = \begin{cases} (a - g(x, y_j))y_j, & \text{if } y_j \le \xi \\ (b - g(x, y_j))y_j + (a - b)\xi, & \text{if } y_j \le \xi, \end{cases}$$

in which

$$g(x, y_j) = x_i, \text{ if } N_i \succ y_j \succ N_i - 1,$$

where $[N_i, N_i - 1]$ means the interval of quantity, $N_0 = 0$ and χ_i means the wholesale price during quantity interval.

If the manufacturer gives the retailers whole price f-1- lowing the incremental discount method, the objective functions can be formulated as follows,

$$F(x, y_1, y_2, ..., y_n) = \sum_{j=1}^{n} [g(x, y_j) - d \times y_j]$$

in which

$$g(x, y_j) = \sum_{k=1}^{i-1} xk \times (N_k - N_{k-1}) + x_i \times (y_j - N_{i-1}).$$

If $N_i \succ y_j \succ N_{i-1}$, where $[N_i, N_i - 1]$ means the interval of quantity, $i = 1, 2, ..., m$ and $N_0 = 0$.

Let

$$g(N_{i-1},\, x_i) = \sum_{k=1}^{i-1} xk \times (N_k - N_{k-1}),.$$

Then

$$g(x_i,\, y_j) = g(N_{i-1},\, x_i) + x_i \times (y_j - N_{I-1}),$$

and

$$f_j(x_i,\, y_j,\, \xi_j)$$
$$= (a+b)\min\{y_j,\, \xi_j\} - g(N_{i-1},\, x_i) + x_i \times (y_j - N_{i-1}) + b \times x_i.$$

4. Methods to Bi-level Programming: Hybrid Intelligent Algorithm

As bi-level programming is an NP-hard problem which has been showed by Ben-Ayed and Blair [6] via the well-known Knapsack Problem. In order to solve the bi-level programming, a lot of numerical algorithm [5], [10], [8] and [9] have been developed. In this paper, a hybrid intelligent algorithm based on simulation and genetic algorithm is developed to solve these models.

In these bi-level programming models, the objective functions of retailers are the expected values of random variable. We can employ stochastic simulation which was given by Lin [7] to compute them.

Algorithm: Stochastic Simulation

Step 1: Set $L = 0$.

Step 2: Generate u according to the distribution function Φ.

Step 3: $L = L + f(u)$.

Step 4: Repeat the second and third steps N times.

Step 5: $L = L/N$.

Genetic Algorithm is a very useful tool to many complex optimization problem. In the hybrid intelligent algorithm, we employ GA to solve optimization of bi-level newsboy problem.

We now have the following main process: first, the manufacturer determines x, after that retailers determine y_j according to x so that maximize their own profit, and return the y_j to manufacturer, the manufacturer can compute his profit according x and y. When the upper level gives the value of x, lower level can get the maximum profit by the following algorithm.

Algorithm for lower level

Step 1: Initialize chromosomes of lower level randomly and check their feasibility.

Step 2: Update the chromosomes of lower level by crossover and mutation operations and check their feasibility.

Step 3: Calculate the objective functions values of lower level for all chromosomes according to above stochastic simulation.

Step 4: Compute the fitness of each chromosome according to the objective values.

Step 5: Select the chromosomes of lower level by spinning the roulette wheel.

Step 6: Repeat the step 2-4 until a terminal condition is satisfied.

Step 7: Return the best chromosome of lower level to the upper level.

With the returning chromosome of lower level, upper level can be solved by genetic algorithm. We will give some numerical examples to show the effectiveness of this algorithm in the following section.

5. Numerical Examples

Now, we consider a bi-level programming with two retailers. The parameters are given as follows:

$$M = 1,\ R = 5,\ u_1 = 100,\ \sigma_1 = 25,\ u_2 = 100,\ \sigma_1 = 35.$$

When the quantity is during different interval, the function of $g(x, y)$ is as follows

$$g(x,\ y) = \begin{cases} x_1, & if\ \ 0 \prec y \le 50, \\ x_2, & if\ \ 50 \prec y \le 100, \\ x_3, & if\ \ y \succ 100. \end{cases}$$

The hybrid algorithm (2000 cycles in simulation, 200 generations in GA) is employed to solve the model. When the manufacturer gives whole price with all-unit discount and incremental discount, the profit of manufacturer is 425.32 and 427.28 respectively.

6. Conclusion

In this paper, several bi-level programming, model for newsboy problem have been presented, in which multiple retailers and two discount based on GA and simulation is given to solve them. In order to illustrate the effectiveness of this algorithm, some numerical examples are presented.

References

[1] Lin Chen-Sin, The single-item newsboy problem with dual performance measures and quantity discounts, European Journal of Operational Research 100 (1997), 562-565.

[2] Khouja, the newsboy problem under progressive multiple discounts, European Journal of Operational Research 84 (1995), 458-466.

[3] A. H. L. Lau, Some two-echelon style-goods inventory models with asymmetric market information, European Journal of Operational Research 134 (2001), 29-42.

[4] A. H. L. Lau, Comparative normative optimal behavior in two echelon multiple retailer distribution systems for a single period product, European Journal of Operational Research 144 (2003), 659-676.

[5] B. Liu, Stackelberg-Nash equilibrium for multilevel programming with multiple followers using genetic algorithms, Comput. Math. Appl. 36(7) (1998), 79-89.

[6] O. Ben-Ayed and C. E. Blair, Computational difficulties of bi-level linear programming, Operations Research 38 (1990), 556-560.

[7] B. Liu, Theory and Practice of Uncertain Programming, Physics-Verlag, Heidelberg, 2002.

[8] J. Gao, Stochastic Bilevel Programming Problem, Technical Report, 2002.

[9] Y. Yin, Genetic-algorithms-based approach for bi-level programming models, T. Journal of Thransportation Engineeing 126 (2000), 115-120.

[10] K. Shimizu and E. Aiyoshi, A new computational method for stackelberg and minmax problems by use a penalty method, IEEE Transactions on Automatic Control AC-26 (1981), 460-466.